SMOKE PRODUCTION
AND PROPERTIES

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INTRODUCTION

The term "smoke" is defined in this chapter as the smoke aerosol or condensed phase component of the products of combustion. This differs from the American Society for Testing and Materials (ASTM) definition of smoke, which includes the evolved gases as well. Smoke aerosols vary widely in appearance and structure, from light colored, for droplets produced during smoldering combustion and fuel pyrolysis, to black, for solid, carbonaceous particulate or soot produced during flaming combustion. A large fraction of the radiant energy emitted from a fire results from the blackbody emission from the soot in the flame. The subject of radiant heat transfer is of such importance that it is treated in a separate chapter. This chapter focuses on smoke aerosols outside the combustion zone.

The effects of the smoke produced by a fire depend on the amount of smoke produced and on the properties of the smoke. The following section presents experimental results on smoke emission for a variety of materials. The smoke emission, together with the flow pattern, determines the smoke concentration as smoke moves throughout a building.

The most basic physical property of smoke is the size distribution of its particles. Results on size distribution for various types of smoke and techniques used for measuring particle size are presented in the section "Size Distribution." The section "Smoke Properties" focuses on those properties of greatest concern to the fire protection community: light extinction coefficient of smoke, visibility through smoke, and detectability of smoke. These properties are primarily determined by the smoke concentration and the particle size distribution. References for other smoke aerosol properties, such as diffusion coefficient and sedimentation velocity, are also provided.

SMOKE PRODUCTION

Smoke emission is one of the basic elements for characterizing a fire environment. The combustion conditions under which smoke is produced—flaming, pyrolysis, and smoldering—affect the amount and character of the smoke. The smoke emission from a flame represents a balance between growth processes in the fuel-rich portion of the flame and burnout with oxygen. While it is not possible at the present time to predict the smoke emission as a function of fuel chemistry and combustion conditions, it is known that an aromatic polymer, such as polystyrene, produces more smoke than hydrocarbons with single carbon-carbon bonds, such as polypropylene. The smoke produced in flaming combustion tends to have a large content of elemental (graphitic) carbon.

Pyrolysis occurs at a fuel surface as a result of an elevated temperature; this may be due to a radiant flux heating the surface. The temperature of a pyrolyzing sample, 600 to 900 K, is much less than the gas phase flame temperature, 1200 to 1700 K. The vapor evolving from the surface may include fuel monomer, partially oxidized products, and polymer chains. As the vapor rises, the low vapor pressure constituents can condense, forming smoke droplets appearing as light-colored smoke.

Smoldering combustion also produces smoke droplets, but in this case the combustion is self-sustaining, whereas pyrolysis requires an external heat source. While most materials can be pyrolyzed, only a few materials, including cellulosic materials (wood, paper, cardboard, etc.) and flexible polyurethane foam, are able to smolder. The temperature during smoldering is typically 600 to 1100 K.

In Table 2-15.1 the smoke conversion factor, ε, is given for a variety of materials commonly found in buildings. The quantity ε is defined as the mass of smoke produced/mass of fuel burned.

The references cited in Table 2-15.1 should be consulted regarding the detailed description of the combustion conditions. In many instances, ε was measured for a range of radiant fluxes, oxygen concentrations, sample orientations, and ambient temperatures. It is seen in Table 2-15.1 that ε has a greater range for flaming combustion, with values in the range 0.001 to 0.17, compared to pyrolysis and smoldering, with values in the range 0.01 to 0.17. The following factors should be taken into account when using this table for smoke emission estimates:

1. Most of the measurements reported in Table 2-15.1 were made on small-scale samples.
### TABLE 2-15.1 Smoke Production for Wood and Plastics

<table>
<thead>
<tr>
<th>Type</th>
<th>Smoke Conversion Factor, $\varepsilon$</th>
<th>Combustion Conditions</th>
<th>Fuel Area, m²</th>
<th>Ref. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Douglas fir</td>
<td>0.03–0.17</td>
<td>pyrolysis</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>&lt;0.01–0.025</td>
<td>flaming</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>hardboard</td>
<td>0.0004–0.001</td>
<td>flaming*</td>
<td>0.0005</td>
<td>2</td>
</tr>
<tr>
<td>fiberboard</td>
<td>0.005–0.01</td>
<td>flaming*</td>
<td>0.0005</td>
<td>2</td>
</tr>
<tr>
<td>polyvinylchloride</td>
<td>0.03–0.12</td>
<td>pyrolysis</td>
<td>0.005</td>
<td>3</td>
</tr>
<tr>
<td>polyvinylchloride</td>
<td>0.12</td>
<td>flaming</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polyurethane (flexible)</td>
<td>0.07–0.15</td>
<td>pyrolysis</td>
<td>0.005</td>
<td>3</td>
</tr>
<tr>
<td>polyurethane (flexible)</td>
<td>&lt;0.01–0.035</td>
<td>flaming</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polyurethane (rigid)</td>
<td>0.06–0.19</td>
<td>pyrolysis</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polyurethane (rigid)</td>
<td>0.09</td>
<td>flaming</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.17 ($m_{O_2} = 0.30$)**</td>
<td>flaming</td>
<td>0.0005</td>
<td>4</td>
</tr>
<tr>
<td>polystyrene</td>
<td>0.15 ($m_{O_2} = 0.23$)</td>
<td>flaming</td>
<td>0.07</td>
<td>5</td>
</tr>
<tr>
<td>polypropylene</td>
<td>0.12</td>
<td>pyrolysis</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polypropylene</td>
<td>0.016</td>
<td>flaming</td>
<td>0.005</td>
<td>1</td>
</tr>
<tr>
<td>polypropylene</td>
<td>0.08 ($m_{O_2} = 0.23$)</td>
<td>flaming</td>
<td>0.007</td>
<td>5</td>
</tr>
<tr>
<td>polypropylene</td>
<td>0.10 ($m_{O_2} = 0.23$)</td>
<td>flaming</td>
<td>0.07</td>
<td>5</td>
</tr>
<tr>
<td>polymethylmethacrylate</td>
<td>0.02 ($m_{O_2} = 0.23$)</td>
<td>flaming</td>
<td>0.07</td>
<td>5</td>
</tr>
<tr>
<td>polyoxymethylene</td>
<td>~0</td>
<td>pyrolysis</td>
<td>0.007</td>
<td>5</td>
</tr>
<tr>
<td>cellulosic insulation</td>
<td>0.01–0.12</td>
<td>smoldering</td>
<td>0.02</td>
<td>6</td>
</tr>
</tbody>
</table>

*Sample smoldered for a period of time after the pilot flame was extinguished.

**$m_{O_2}$ refers to mol fraction of O₂.

2. Most experiments were for free burning at ambient conditions; reduced ventilation can strongly affect the smoke production.

3. In transport, the smoke may coagulate, partially evaporate, and deposit on surfaces through diffusion and sedimentation. Also, additional smoke may be formed through condensation.

### SIZE DISTRIBUTION

Smoke particle size distribution, together with the amount of smoke produced, primarily determines the properties of the smoke. A widely used representation of the size distribution is the geometric number distribution, $\Delta N / \Delta \log d$, versus $\log d$, where $d$ represents the particle diameter. The quantity $\Delta N$ represents the number of particles per cm³, with diameter between $\log d$ and $\log d + \Delta \log d$. As an example, the particle size distribution of smoke produced by a smoldering incense stick is plotted in Figure 2-15.1, where $\Delta \log d$ for each discrete size range equals 0.25. In this case, the total number concentration for a given size range equals $0.25(\Delta N / \Delta \log d)$. It is seen that the logarithmic scale is necessary by the wide range in particle size and concentration.

For many applications, the most important characteristics of a size distribution are the average particle size and the width of the distribution. A widely used measure of the average size is the geometric mean number diameter, $d_{gm}$, defined by

$$
\log d_{gm} = \frac{1}{N} \sum_{i=1}^{N} N_i \log d_i
$$

where $N$ is the total number concentration, $N_i$ is the number concentration in the $i$th interval, and $\log$ is to the base 10. For the size distribution plotted in Figure 2-15.1, $d_{gm} = 0.072$ μm.

The corresponding measure of the width of the size distribution is the geometric standard deviation, $\sigma_g$. 

![Fig. 2-15.1. Size distribution of incense smoke as measured by an electrical aerosol analyzer. There is a large uncertainty in the dashed portion of the curve.](image-url)
\[ \log \sigma_g = \left[ \frac{\sum (\log d_i - \log d_{gn})^2 N_i}{N} \right]^{1/2} \]  

For the size distribution plotted in Figure 2-15.1, \( \sigma_g = 1.75 \). A perfectly monodisperse distribution would correspond to \( \sigma_g = 1 \). The parameters \( d_{gn} \) and \( \sigma_g \) are useful because actual size distributions are observed to be approximately log-normal, which is the same as a normal or Gaussian distribution, except that \( \log d \) is normally distributed instead of \( d \). An important characteristic of the log-normal distribution is that 68.3 percent of the total particles are in the size range \( \log d_{gn} \leq \log \sigma_g \); for \( d_{gn} = 0.072 \ \mu m \) and \( \sigma_g = 1.75 \), this corresponds to the size range of 0.041 to 0.126 \( \mu m \).

**EXAMPLE 1:**

Compute \( d_{gn} \) and \( \sigma_g \) for the data given below:

<table>
<thead>
<tr>
<th>Interval, ( \mu m )</th>
<th>( d_i )</th>
<th>( N_i, \ cm^{-3} )</th>
<th>( \log d_i )</th>
<th>( \log d_i, \ cm^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0056-0.010</td>
<td>0.0078</td>
<td>6 \times 10^4</td>
<td>-2.11</td>
<td>-1.27 \times 10^5</td>
</tr>
<tr>
<td>0.010-0.018</td>
<td>0.014</td>
<td>2 \times 10^5</td>
<td>-1.85</td>
<td>-3.7 \times 10^5</td>
</tr>
<tr>
<td>0.018-0.032</td>
<td>0.025</td>
<td>4 \times 10^5</td>
<td>-1.60</td>
<td>-6.4 \times 10^5</td>
</tr>
<tr>
<td>0.032-0.056</td>
<td>0.044</td>
<td>9 \times 10^4</td>
<td>-1.26</td>
<td>-1.22 \times 10^5</td>
</tr>
<tr>
<td>0.056-0.10</td>
<td>0.078</td>
<td>3 \times 10^4</td>
<td>-1.11</td>
<td>-3.33 \times 10^4</td>
</tr>
<tr>
<td>0.10-0.18</td>
<td>0.14</td>
<td>1 \times 10^3</td>
<td>-0.85</td>
<td>-0.85 \times 10^3</td>
</tr>
</tbody>
</table>

\[ \log d_{gn} = \log \left( \frac{3.87 \times 10^4}{7.81 \times 10^5} \right)^{1/2} = 0.022 \ \mu m \]

Compute the geometric standard deviation:

\[ \sigma_g = 10^{\left(\frac{3.87 \times 10^4}{7.81 \times 10^5}\right)^{0.5}} = 1.67 \]

The size distribution plotted in Figure 2-15.1 is based on electrical mobility analysis of the smoke aerosol. Figures 2-15.2 and 2-15.3 show size distributions of droplet smoke produced by smoldering cellulosic insulation, as measured by an optical particle counter and by two cascade impactors. The smoke volume distribution plotted in Figure 2-15.3 for the optical particle counter is obtained from the number distribution, using the following relation

\[ V_i = N_i \frac{1}{6} \pi d_i^3 \]  

For particles sized above 1 \( \mu m \), impactors provide more reliable information on the smoke volume distribution than optical particle counters. An optical particle counter is the preferred instrument for the number distribution measurement.
special problems because of the high concentration, wide particle size range, and sometimes high temperature. In selecting an instrument it is important to make the following considerations:

1. Will the instrument respond to the smoke of interest? For example, the piezoelectric mass monitor does not respond well to soot.
2. Will dilution of the smoke be required?
3. Is the measurement size range of the instrument adequate?
4. Is a mass or number distribution measurement appropriate?
5. What is the particle size resolution needed?
6. Is real-time measurement capability needed?
7. Will the instrument perform at the temperature of the smoke environment?

In Table 2-15.3, average particle size and the width of the size distribution are presented for smoke generated by a variety of materials. The results are most meaningful for smoke droplets produced during pyrolyzing and smoldering combustion. In the case of flaming combustion, complex soot agglomerates are formed as shown in Figure 2-15.4. For soot agglomerates, the apparent particle size depends on the measurement technique, unlike the case for spherical smoke droplets.

Smoke aerosols are dynamic with respect to their particle size distribution function. Smoke particles or droplets undergoing Brownian motion collide and stick together. The result of this behavior is that, in a fixed volume of smoke-laden gas, the number of particles decreases while the total mass of the aerosol remains unchanged. This process is known as coagulation. The fundamental parameter for describing coagulation is the coagulation coefficient, \( \Gamma \), the rate constant for the coagulation equation

\[
\frac{dN}{dt} = -\Gamma N^2
\]

For smoke produced from incense sticks, \( \Gamma \) was found to be about \( 4 \times 10^{-10} \text{ cm}^3/\text{s} \) and about \( 1 \times 10^{-9} \text{ cm}^3/\text{s} \) for smoke

<table>
<thead>
<tr>
<th>Instrument Type</th>
<th>Function/Range</th>
<th>Advantage/Limitation for Smoke Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>filter-collection</td>
<td>mass conc.</td>
<td>accurate, slow</td>
</tr>
<tr>
<td>piezoelectric mass monitor</td>
<td>mass conc.</td>
<td>real-time output, but dilution required if &gt;20 mg/m³; does not respond well to soot</td>
</tr>
<tr>
<td>tapered element oscillating microbalance</td>
<td>0.01 &lt; d &lt; 5 ( \mu ) m</td>
<td>real time, 0.1–1000 mg/m³; replace filter after 3–100 mg deposit</td>
</tr>
<tr>
<td>condensation nuclei counter</td>
<td>mass conc.</td>
<td>&lt;5 ( \mu ) m</td>
</tr>
<tr>
<td></td>
<td>0.005 &lt; d &lt; 2 ( \mu ) m</td>
<td>&lt;3 \times 10⁵ particles/cm³</td>
</tr>
<tr>
<td>photometer</td>
<td>scattered light</td>
<td>1.1–1000 mg/m³</td>
</tr>
<tr>
<td>nephelometer</td>
<td>0.1–10 ( \mu ) m</td>
<td>&lt;5 mg/m³</td>
</tr>
<tr>
<td>electrical aerosol analyzer</td>
<td>total light scattered size distribution</td>
<td>&lt;5 \times 10⁵ particles/cm³; 2 min/scan</td>
</tr>
<tr>
<td>cascade impactor</td>
<td>mass size distribution</td>
<td>no dilution needed, can be used at high temp., large sample required</td>
</tr>
<tr>
<td>optical particle counter</td>
<td>number distribution</td>
<td>highest resolution, &lt;10⁵ particles/cm³, large dilution</td>
</tr>
</tbody>
</table>

*Low-pressure impactor extends size range down to 0.05 \( \mu \) m.
**Laser model extends size range down to 0.1 \( \mu \) m and concentration up to 10⁶ particles/cm³.
TABLE 2-15.3 Particle Size of Smoke from Burning Wood and Plastics

<table>
<thead>
<tr>
<th>Type</th>
<th>( d_{\text{gm}}, \ \mu\text{m}^* )</th>
<th>( d_{\text{32}}, \ \mu\text{m}^{**} )</th>
<th>( \sigma_g )</th>
<th>Combustion Conditions</th>
<th>Ref. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Douglas fir</td>
<td>0.5–0.9</td>
<td>0.75–0.8</td>
<td>2.0</td>
<td>pyrolysis</td>
<td>1, 3</td>
</tr>
<tr>
<td>Douglas fir</td>
<td>0.43</td>
<td>0.47–0.52</td>
<td>2.4</td>
<td>flaming</td>
<td>1, 3</td>
</tr>
<tr>
<td>polyvinylchloride</td>
<td>0.9–1.4</td>
<td>0.8–1.1</td>
<td>1.8</td>
<td>pyrolysis</td>
<td>3</td>
</tr>
<tr>
<td>polyvinylchloride</td>
<td>0.4</td>
<td>0.3–0.6</td>
<td>2.2</td>
<td>flaming</td>
<td>3</td>
</tr>
<tr>
<td>polyurethane (flexible)</td>
<td>0.8–1.8</td>
<td>0.8–1.0</td>
<td>1.8</td>
<td>pyrolysis</td>
<td>3</td>
</tr>
<tr>
<td>polyurethane (flexible)</td>
<td>0.5–0.7</td>
<td>1.0</td>
<td>2.3</td>
<td>pyrolysis</td>
<td>3</td>
</tr>
<tr>
<td>polyurethane (rigid)</td>
<td>0.3–1.2</td>
<td>0.6</td>
<td>1.9</td>
<td>pyrolysis</td>
<td>3</td>
</tr>
<tr>
<td>polystyrene</td>
<td>1.4</td>
<td></td>
<td></td>
<td>pyrolysis</td>
<td>1</td>
</tr>
<tr>
<td>polystyrene</td>
<td>1.3</td>
<td></td>
<td></td>
<td>flaming</td>
<td>1</td>
</tr>
<tr>
<td>polypropylene</td>
<td>1.6</td>
<td></td>
<td></td>
<td>pyrolysis</td>
<td>1</td>
</tr>
<tr>
<td>polypropylene</td>
<td>1.2</td>
<td></td>
<td></td>
<td>flaming</td>
<td>1</td>
</tr>
<tr>
<td>polymethylmethacrylate</td>
<td>0.6</td>
<td></td>
<td></td>
<td>pyrolysis</td>
<td>1</td>
</tr>
<tr>
<td>polymethylmethacrylate</td>
<td>1.2</td>
<td></td>
<td></td>
<td>flaming</td>
<td>1</td>
</tr>
<tr>
<td>cellulosic insulation</td>
<td>2–3</td>
<td></td>
<td></td>
<td>smoldering</td>
<td>6</td>
</tr>
</tbody>
</table>

*\( d_{\text{gm}} \) is analogous to \( d_{\text{gm}} \) but with mass replacing volume in Equation 4. Values of \( d_{\text{gm}} \) less than about 0.5 \( \mu\text{m} \) are probably overestimated arising from the minimum size resolution of the impactor at about 0.4 \( \mu\text{m} \).

**The quantity \( d_{\text{32}} \) is obtained by optical measurements:

\[
\sigma_{32} = \frac{\sum N_i d_i^2}{\sum N_i}.
\]

The smoke properties of primary interest to the fire community are light extinction, visibility, and detection. For completeness, a list of other smoke aerosol properties and references is given in Table 2-15.4.

**EXAMPLE 2:**

Calculate the change in the number concentration over a 5 min time interval for a uniformly distributed smoke, generated from flaming a-cellulose given an initial concentration of \( 1 \times 10^7 \) particles/cm³.

Integrating Equation 6, yields

\[
N = \frac{N_0}{1 + \Gamma N_0 t} = \frac{1 \times 10^7}{1 + (10^{-9})(10^7)(300)} = \frac{10^7}{1 + 3}
\]

\[
N = 2.5 \times 10^6 \text{ particles/cm}^3
\]

So in this example, there is a fourfold reduction in number concentration due to coagulation.

The effect of the decrease in number concentration on the size distribution is treated by Mulholland et al. A general discussion of coagulation phenomena in aerosols is given by Friedlander. In addition to coagulation, other smoke-aging processes, including condensation of vapor onto existing particles and evaporation of the volatile component of the smoke, can also take place. There is relatively little information on these processes. Also, smoke particles can be lost to the walls, ceiling, and floor of an enclosure through a variety of processes, including diffusion, sedimentation, and thermophoresis.

**SMOKE PROPERTIES**

The smoke properties of primary interest to the fire community are light extinction, visibility, and detection. For completeness, a list of other smoke aerosol properties and references is given in Table 2-15.4.

The most widely measured smoke property is the light extinction coefficient. The physical basis for light extinction measurements is Bouguer's law, which relates the intensity, \( I_0 \), of the incident monochromatic light of wavelength \( \lambda \) and the intensity of the light, \( I_\lambda \), transmitted through pathlength, \( L \), of the smoke.

**Fig. 2-15.4. Transmission electron micrograph of a soot particle. The overall size of the agglomerate is about 6 \( \mu\text{m} \), and the diameter of the individual spheres is about 0.03 \( \mu\text{m} \).**
\[
\frac{I_0}{I_\lambda} = e^{-KL} \tag{7}
\]

where \(K\) is the light extinction coefficient. When Equation 7 is expressed in terms of base 10

\[
\frac{I_0}{I_\lambda} = 10^{-DL} \tag{8}
\]

The quantity \(D\) is defined as the optical density per meter, and \(D = K/2.3\).

The extinction coefficient, \(K\), is an extensive property and can be expressed as the product of an extinction coefficient per unit mass, \(K_m\), and mass concentration of the smoke aerosol, \(m\).

\[
K = K_mm \tag{9}
\]

The specific extinction coefficient, \(K_m\), depends on the size distribution and optical properties of the smoke through the relation

\[
K_m = \frac{3}{2\rho m} \int_{d_{min}}^{d_{max}} \frac{1}{d} \frac{\delta m}{\delta d} Q_{ext}(d/\lambda, n_r) \delta d \tag{10}
\]

In Equation 10 the symbol \(\delta m/\delta d\) represents the mass size distribution. The single particle extinction efficiency, \(Q_{ext}\), is a function of the ratio of particle diameter to wavelength of light, \(d/\lambda\), and of the complex refractive index of the particle, \(n_r\). The quantity \(p\) represents the particle density.

Seader and Einhorn\(^{11}\) obtained \(K_m\) values of 7.6 m\(^2\)/g for smoke produced during flaming combustion of wood and plastics and a value of 4.4 m\(^2\)/g for smoke produced during pyrolysis of these materials. The experiments were small scale, utilizing samples of about 50 cm\(^2\), and the value of \(K_m\) represents an integrated result over the test. The light source used in the measurements was polychromatic, while Bouguer's law is strictly valid only for monochromatic light. Foster\(^{12}\) predicted a 22 percent deviation from Bouguer's law over the mass concentration range from 0.06 to 2.8 g/m\(^3\) as a result of using a polychromatic light source with wood smoke. Still, it is useful to use the Seader and Einhorn\(^{11}\) result as a rough guide if more detailed optical data on the smoke of interest is not available.

Mulholland\(^{13}\) has described the general design of a light extinction instrument that satisfies Bouguer's law. Two key features are the use of monochromatic light and the elimination of forward scattered light at the detector.

The specific optical density, \(D_m\), is measured in a standard laboratory smoke test\(^{14}\) for assessing the amount of visible smoke produced in a fire. The dimensionless quantity \(D_m\) is defined by

\[
D_m = \frac{DV_c}{A} \tag{11}
\]

where \(V_c\) is the volume of the chamber, and \(A\) is the area of the sample. This is a convenient quantity to measure if the decomposed area is well defined. Since \(D_m\) depends on the sample thickness, the same thickness should be used for relative rating of materials tested. Table 2-15.5 includes results for \(D_m\) based on small-scale experiments with wood and plastics by Gross et al.\(^{14}\), Seader and Chien,\(^{15}\) and Breden and Meisters.\(^{16}\) Lopez\(^{17}\) demonstrated a correlation for \(D_m\) between small- and large-scale fires for aircraft interior construction materials.

If the mass loss of the sample is measured, then the mass optical density, \(D_m\), is the appropriate measure of visible smoke.

\[
D_m = \frac{DV_c}{\Delta M} \tag{12}
\]

This technique requires an accurate measurement of the mass loss of the sample, \(\Delta M\), in addition to a light extinction measurement. Table 2-15.5 includes results for \(D_m\) for a variety of materials studied by Seader and Chien,\(^{15}\) Breden and Meisters,\(^{16}\) Babrauskas,\(^{16}\) and Evans.\(^{19}\) The results of Babrauskas' study were expressed in terms of \(D_m\) by Quintiere.\(^{20}\)

In two of the studies,\(^{18,19}\) a comparison was made between \(D_m\) measured in small-scale tests and \(D_m\) measured in large-scale tests. The large-scale tests involved mattresses\(^{16}\) in one case and plastic utility tables\(^{18}\) in the other. In these two cases, there appeared to be a qualitative correlation between \(D_m\) measured for small- and large-scale tests. Quintiere\(^{20}\) has made an extensive investigation of the correlation between small- and large-scale studies in terms of \(D_m\) and \(D_s\) and finds that the correlation breaks down as fires become more complex. From his review of the literature, Quintiere\(^{20}\) suggests that heat flux and ventilation conditions can have a major effect on smoke production.

In most cases of practical interest, an important goal is to be able to predict the extinction coefficient based on information regarding \(D_s\) or \(D_m\). The extinction coefficient, in turn, is related to visibility through the smoke, as discussed below.

### Visibility

Visibility of exit signs, doors, and windows can be of great importance to an individual attempting to survive a fire. To see an object requires a certain level of contrast between the object and its background. For an isolated object surrounded by a uniform, extended background, contrast, \(C\), can be defined as\(^{21}\)

\[
C = \frac{B}{B_0} - 1 \tag{13}
\]

where \(B\) is the brightness or luminance of the object, and \(B_0\) is the luminance of the background. For daylight conditions, with a black object being viewed against a white background, a value of \(C = -0.02\) is often used as the contrast threshold at which an object can be discerned against the background. The visibility of the object, \(S\), is the distance at which the contrast is reduced to \(-0.02\). Most visibility measurements through smoke have relied on test subjects to determine the distance at which the object was no longer visible rather than the actual measurement of \(C\) with a photometer.

---

**TABLE 2-15.4 Smoke Aerosol Properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Ref. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>diffusion coefficient</td>
<td>8</td>
</tr>
<tr>
<td>sedimentation velocity</td>
<td>9</td>
</tr>
<tr>
<td>thermophoretic velocity</td>
<td>10</td>
</tr>
<tr>
<td>aerodynamic diameter</td>
<td>9</td>
</tr>
<tr>
<td>electrical mobility</td>
<td>9</td>
</tr>
<tr>
<td>thermal charging</td>
<td>9</td>
</tr>
<tr>
<td>scattering coefficient</td>
<td>8</td>
</tr>
<tr>
<td>extinction coefficient</td>
<td>8</td>
</tr>
<tr>
<td>condensation/evaporation</td>
<td>8</td>
</tr>
</tbody>
</table>
Visibility depends on many factors, including the scattering and the absorption coefficient of the smoke, the illumination in the room, whether the sign is light-emitting or light-reflecting, and the wavelength of the light. Visibility also depends on the individual’s visual acuity and on whether the eyes are “dark-adapted’” or “light-adapted.” Nevertheless, a fair correlation between visibility of test subjects and the extinction coefficient of the smoke has been obtained in an extensive study by Jin22 as illustrated in Figure 2-15.5. The visibility of light-emitting signs was found to be two to

<table>
<thead>
<tr>
<th>Type (Sample #)</th>
<th>Maximum $D_m$</th>
<th>$D_m$ ( (m^2/g) )</th>
<th>Combustion Conditions</th>
<th>Sample* Thickness (cm)</th>
<th>Ref. No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>hardboard</td>
<td>6.7 $\times 10^1$</td>
<td>flaming</td>
<td>0.5</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>hardboard</td>
<td>6.0 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>plywood</td>
<td>1.1 $\times 10^2$</td>
<td>flaming</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>plywood</td>
<td>2.9 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>polystyrene</td>
<td>&gt;660</td>
<td>flaming</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>polystyrene</td>
<td>3.7 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>polystyrene (with plasticizer)</td>
<td>3.0 $\times 10^2$</td>
<td>flaming</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>polystyrene foam</td>
<td>2.0 $\times 10^1$</td>
<td>pyrolysis</td>
<td>1.3</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>polyurethane foam</td>
<td>1.6 $\times 10^1$</td>
<td>flaming</td>
<td>0.8</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>nylon carpet</td>
<td>2.7 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.8</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>nylon carpet</td>
<td>3.2 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.8</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>acrylic</td>
<td>1.1 $\times 10^2$</td>
<td>flaming</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>acrylic</td>
<td>1.6 $\times 10^2$</td>
<td>pyrolysis</td>
<td>0.6</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>plywood</td>
<td>5.3 $\times 10^2$</td>
<td>0.29</td>
<td>0.6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>polymethylmethacrylate</td>
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<td>0.6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>polystyrene</td>
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<td>0.12</td>
<td>0.6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>polystyrene (with plasticizer)</td>
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<td>0.64</td>
<td>0.6</td>
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</tr>
<tr>
<td>neoprene</td>
<td>8.8 $\times 10^2$</td>
<td>0.55</td>
<td>0.6</td>
<td>15</td>
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</tr>
<tr>
<td>Douglas fir</td>
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<td>0.28</td>
<td>0.6</td>
<td>15</td>
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</tr>
<tr>
<td>polypropylene</td>
<td>4.0 $\times 10^2$</td>
<td>0.53</td>
<td>flaming**</td>
<td>0.4</td>
<td>16</td>
</tr>
<tr>
<td>polyethylene</td>
<td>2.9 $\times 10^2$</td>
<td>0.29</td>
<td>flaming**</td>
<td>0.4</td>
<td>16</td>
</tr>
<tr>
<td>paraffin wax</td>
<td>2.3 $\times 10^2$</td>
<td>0.23</td>
<td>flaming**</td>
<td>0.4</td>
<td>16</td>
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<td>polystyrene</td>
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<td>flaming**</td>
<td>0.4</td>
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<td>styrene</td>
<td>0.96</td>
<td>flaming**</td>
<td>0.4</td>
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<tr>
<td>polyvinylchloride</td>
<td>0.34</td>
<td>flaming**</td>
<td>0.4</td>
<td>16</td>
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<tr>
<td>polyoxymethylene</td>
<td>--</td>
<td>flaming**</td>
<td>0.4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>polyurethane (7A)</td>
<td>2.1 $\times 10^2$</td>
<td>flaming</td>
<td>1.3</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>polyurethane (7A)</td>
<td>1.5 $\times 10^2$</td>
<td>flaming***</td>
<td>1.3</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>wool (8A)</td>
<td>&gt;5.5 $\times 10^2$</td>
<td>fluorning***</td>
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<td>17</td>
<td></td>
</tr>
<tr>
<td>wool (8A)</td>
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<td>flourning***</td>
<td>0.9</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>acrylic (9B)</td>
<td>5.8 $\times 10^1$</td>
<td>florning***</td>
<td>0.14</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>acrylic (9B)</td>
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<td>flourning***</td>
<td>0.14</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>polyurethane (MO1)</td>
<td>0.33</td>
<td>fluorning**</td>
<td>--</td>
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<tr>
<td>polyurethane (MO1)</td>
<td>0.22</td>
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<tr>
<td>cotton (MO3)</td>
<td>0.17</td>
<td>fluorning**</td>
<td>--</td>
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<tr>
<td>cotton (MO3)</td>
<td>0.12</td>
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<tr>
<td>latex (MO4)</td>
<td>0.65</td>
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<td>18</td>
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<td>latex (MO4)</td>
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<td>neoprene (MO8)</td>
<td>0.40</td>
<td>fluorning**</td>
<td>--</td>
<td>18</td>
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<tr>
<td>neoprene (MO8)</td>
<td>0.20</td>
<td>fluorning**</td>
<td>--</td>
<td>18</td>
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<tr>
<td>polystyrene (7)</td>
<td>0.79</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>polystyrene (7)</td>
<td>1.0</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>polystyrene foam (16)</td>
<td>0.79</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>polystyrene foam (16)</td>
<td>0.82</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
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</tr>
<tr>
<td>ABS (18)</td>
<td>0.52</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>ABS (18)</td>
<td>0.54</td>
<td>fluorning</td>
<td>--</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>

*Sample area is 0.005 m² in vertical configuration, unless stated otherwise.
**Sample is in horizontal configuration (0.005 m²).
***0.09 m² sample size.
1The value of $D_m$ is computed by Quintiere,20 based on data in Babrauskas.18
2The sample is a mattress.
3The sample is a plastic utility table.
Fig. 2-15.5. Visibility versus extinction coefficient for a light-emitting sign (○) and light-reflecting sign (●). The range bars include data for both flame- and smolder-generated smoke and sign illumination levels varying by about a factor of 4.

The data is based on the subjects viewing the smoke through glass so that the irritant effect of the smoke was eliminated. Jin and Yamada\(^23\) have studied the visual acuity and eye-blink rate for highly irritant white smoke produced by burning wood cribs. They found that the ratio of visual acuity without goggles to acuity with goggles decreases markedly for smoke extinction coefficient, \(K\), greater than 0.25 m\(^{-1}\).

**EXAMPLE 3:**

Estimate the visibility of a light-reflecting exit sign in a 6 m square room with a 2.5-m height, as a result of flaming combustion of a 200-g polyurethane foam pillow.

The smoke yield for flexible polyurethane, according to Table 2-15.1, is about 0.03 for flaming combustion. This implies a smoke emission, \(M_s\), given by

\[
M_s = (0.03)(200) = 6 \text{ g}
\]

The corresponding mass concentration in the room, \(m\), is

\[
m = \frac{6}{(6)^2(2.5)} = 0.067 \text{ g/m}^3
\]

Taking \(K_m\) to be 7.6 m\(^2\)/g for flaming combustion, one obtains \(K\) using Equation 9

\[
K = \frac{7.6}{0.067} = 0.51 \text{ m}^{-1}
\]

The visibility is next estimated using Equation 15

\[
S = \frac{3}{K} = \frac{3}{0.51} = 5.9 \text{ m}
\]

It is important to point out the approximations made in this analysis:

1. The smoke is confined to the room and is well-mixed. Actually the concentration will be higher near the ceiling and decrease abruptly below the flame.
2. The value of 0.03 for the smoke conversion factor, \(e\), is an estimated value in the upper part of the range (0.01 to 0.035) for generic flexible polyurethane foams measured in small-scale experiments and may not be appropriate for a pillow. In a realistic case, the pillow would probably smolder before flaming, and \(e\) is much larger in the smolder mode.
3. The value of \(K_m\) is based on a limited number of small-scale experiments with a polychromatic light source.
4. The range of validity of Equation 15 has not been widely studied.

An alternative method for estimating the visibility is based on using the mass optical density data in Table 2-15.5. The quantity \(D_m\) for the pillow is estimated to be 0.22 m\(^2\)/g based on Babrauskas’ results\(^18\) given in Table 2-15.5 for polyurethane (m 01). On rearranging Equation 12, the following result is obtained

\[
D = \frac{D_m \Delta M}{V_c} = \frac{(0.22)(200)}{(6)^2(2.5)} = 0.49 \text{ m}^{-1}
\]

The smoke extinction coefficient, \(K_s\) is 1.12 m\(^{-1}\) or 2.3 times \(D\). Using Equation 15, we obtain \(S = 2.7 \text{ m}\) compared to 5.9 m obtained by the first method. In principle, the second method is more reliable, because it is more direct.

**Detection**

In addition to their utility for estimating visibility, light extinction measurements are also widely used in characterizing smoke detector performance. Underwriters Laboratories’ (U.L.) acceptance testing of smoke detectors\(^24\) is based in part on a minimum sensitivity based on optical density per meter, \(D\), of 0.06 (4 percent obscuration per ft for a 5 ft beam length) for grey-color (cellulosic) smoke and 0.14 (10 percent per ft) for black smoke (kerosene).

The electrical output of a detector, \(P\), from a light-scattering or ionization-type smoke detector can be represented as an integrated product of the size distribution function and the basic response of the detector, \(R(d)\).

\[
P = \int_{d_{min}}^{d_{max}} R(d) \frac{\Delta M}{8d} \, \text{d}d
\]

The response functions for two smoke detectors are plotted in Figure 2-15.6. It is seen that the ionization-type smoke detector is more sensitive to smoke particles smaller than about 0.3 \(\mu\)m, and the light-scattering type more sensitive to particles larger than 0.3 \(\mu\)m.

The basic principle of ionization detectors is the interception of gaseous ions by smoke particles, reducing the ion current in the detector until a preset alarm point is reached. The detector response function is approximately proportional to the product of the number concentration and particle diameter.\(^{25,26}\) For one detector\(^25\) the response function is given by

\[
R(d) = cd
\]

where \(c\) has a value of 7 in units of \(\mu\)V per particle concentration per \(\mu\)m (\(\mu\)V cm\(^3\)/\(\mu\)m). Such detectors tend to be most sensitive to high concentrations of small particles, such as those produced by flaming paper and wood fires, and least sensitive to the low concentration of large smoke droplets produced in smoldering fires.
The purpose of smoke detectors is to give the occupants of a room adequate warning to escape a developing fire. The final examples of this chapter illustrate how to utilize all the concepts discussed above to estimate escape time.

**EXAMPLE 4:**

Suppose the pillow in the preceding example is burning at a steady rate of 50 g/min. How long would it take for an ionization detector with response function given by Equation 17 to alarm? Assume an alarm voltage of 2.5 V above background. How much time would an individual have before the visibility decreased to an unsafe level?

**SOLUTION:**

First consider a first principle analysis based on the size distribution of the smoke. From Equations 16 and 17

\[ P = c \int_0^{d_{max}} dN \frac{\delta d}{\delta d} \delta \]  

The following three identities for the log-normal distribution are needed:

\[ \int_0^\infty dN \frac{\delta N}{\delta d} \delta d = N_0 d_{gn} \exp \left( \frac{1}{2} \ln^2 \sigma_g \right) \]  

\[ d_{32} = \frac{\int_0^\infty d^2 N \frac{\delta N}{\delta d} \delta d}{\int_0^\infty dN \frac{\delta N}{\delta d} \delta d} = d_{gn} \exp \left( \frac{5}{2} \ln^2 \sigma_g \right) \]  

\[ \int_0^\infty d^3 N \frac{\delta N}{\delta d} \delta d = N_0 d_{gn}^3 \exp \left( \frac{1}{2} \ln^2 \sigma_g \right) \]  

Here \( N_0 \) refers to the number concentration. Taking \( \delta N/\delta d \) to be log-normal and using Equation 1

\[ P = cN_0 d_{gn} \exp \left( \frac{1}{2} \ln^2 \sigma_g \right) \]

Estimating \( \sigma_g \) to be 2.0 and \( d_{32} \) to be 0.6 \( \mu \)m from Table 2-15.3 for flexible polyurethane, \( d_{gn} \) is determined using Equation 1-2

\[ d_{gn} = d_{32} \exp \left( -\frac{5}{2} \ln^2 \sigma_g \right) = 0.6 \exp \left[ -\frac{5}{2} \left( 0.69 \right)^2 \right] \]

\[ d_{gn} = 0.18 \mu \text{m} \]

Substituting for \( d_{gn} \) and for \( c \) in the expression for \( P \) yields

\[ P = cN_0 d_{gn} \exp \left( \frac{1}{2} \ln \sigma_g^2 \right) = 7N_0 \exp \left( \frac{1}{2} \ln \sigma_g^2 \right) \]

\[ P = 1.6 N_0 \]

The final task is to estimate \( N_0 \) based on the mass generation rate of smoke. In one minute, 50 g of the pillow is consumed and 1.5 g of smoke are produced. This corresponds to a mass concentration, \( m \), given by

\[ m = \frac{1.5}{(6 \times 2.5)} = 0.0167 \text{ g/m}^3 = 1.67 \times 10^{-8} \text{ g/cm}^3 \]

The quantity \( m \) is the third moment of the size distribution

\[ m = \int_0^\infty \frac{1}{6} \pi d^3 N \frac{\delta N}{\delta d} \delta d \]

Using Equation 3,

\[ m = \frac{1}{6} \pi N_0 d_{gn}^2 \exp \left( \frac{1}{2} \ln^2 \sigma_g \right) \]
Finally, solving for $N_0$,

$$N_0 = \frac{6m}{\pi \rho \Delta S} \exp\left(-\frac{3}{2} \ln^2 \sigma_S\right)$$

$$= \frac{(6)(1.67 \times 10^{-6})}{(3.14)(2)(1.8 \times 10^{-5})} \exp\left[-\frac{3}{2} \ln^2 (2.0)\right]$$

$$N_0 = 1.3 \times 10^6 \text{ particles/cm}^3 \text{ (assuming } \rho = 2 \text{ g/cm}^3\text{)}$$

Substituting in the expression for $P$,

$$P = (1.6)(1.3 \times 10^6) = 2.1 \times 10^6 \mu \text{V} = 2.1 \text{ volts}$$

This represents the voltage after 1 min. The estimated time to reach the alarm point, 2.5 V, will be 1.2 min. By the time the entire pillow is consumed in 4 min, the visibility has deteriorated to the point where escape is becoming less likely (visibility 5.9 m, according to Example 3, for a room 6 m across). So the individual’s escape time is:

escape time = time to unsafe condition minus time to detector alarm

$$= 4 \text{ minutes} - 1.2 \text{ minutes} = 2.8 \text{ minutes}$$

Example 4 is intended to illustrate the complete method for estimating the alarm time of smoke detectors. However, there is not adequate information at this time to implement the method in a realistic manner. Information is lacking on the size distribution of smokes and on the detector response functions. The time for the smoke to reach the detector and the time lag for the smoke to enter the sensing zone of the detector are not included in this example, but should be included in a full analysis of the problem.

A simpler method for estimating the alarm time is to calculate the time at which the optical density per meter of the smoke exceeds the value of 0.06 (grey smoke) or 0.14 (black smoke), which correspond to the U.L. minimum sensitivity values. The limitation of this procedure is that a type of smoke may not respond in the same manner to an other with a different size distribution and refractive index.

EXAMPLE 5:

Estimate the time to alarm for the conditions given in Example 4, using the simpler method described above.

SOLUTION:

In Example 3, the optical density was estimated to be 0.49 m$^{-1}$, based on $D_m$ measured for polyurethane. This value corresponds to the burning of the entire pillow. Assuming a steady smoke generation rate, the alarm time [the time at which the minimum detector sensitivity value is exceeded (0.14 for black smoke)] is estimated to be given by

$$t = \frac{0.14}{0.49} (4) = 1.1 \text{ minutes}$$

This is comparable to the estimated 1.2 minutes in Example 4.

NOMENCLATURE

- $d$ volume surface mean diameter, $\mu$m
- $\sigma_S$ geometric standard deviation (–)
- $N$ number concentration, particles/cm$^3$
- $m$ mass concentration of smoke, mg/m$^3$ or g/m$^3$
- $V_T$ volume concentration of smoke, cm$^3$/m$^3$ or $\mu$m$^3$/cm$^3$
- $\Delta N$ number size distribution function, cm$^{-1}$ $\mu$m$^{-1}$
- $\Delta N/d$ geometric number size distribution function, cm$^{-1}$ $\mu$m$^{-1}$
- $\Delta m$ mass size distribution function, mg $\mu$m$^{-1}$ m$^{-3}$
- $\alpha_{ext}$ extinction efficiency (–)
- $\lambda$ wavelength of light, $\mu$m
- $n_r$ complex refractive index of smoke particles
- $K$ extinction coefficient, m$^{-1}$
- $D$ optical density per meter, m$^{-1}$
- $K_m$ specific extinction coefficient, m$^2$/g
- $D_p$ specific optical density (–)
- $V$ volume of chamber, m$^3$
- $M$ mass of smoke, g
- $M_s$ mass of smoke, g
- $d_{32}$ volume surface mean diameter, $\mu$m
- $\sigma_S$ geometric standard deviation (–)
- $N$ number concentration, particles/cm$^3$
- $m$ mass concentration of smoke, mg/m$^3$ or g/m$^3$
- $V_T$ volume concentration of smoke, cm$^3$/m$^3$ or $\mu$m$^3$/cm$^3$
- $\Delta N$ number size distribution function, cm$^{-1}$ $\mu$m$^{-1}$
- $\Delta N/d$ geometric number size distribution function, cm$^{-1}$ $\mu$m$^{-1}$
- $\Delta m$ mass size distribution function, mg $\mu$m$^{-1}$ m$^{-3}$
- $Q_{ext}$ extinction efficiency (–)
- $\lambda$ wavelength of light, $\mu$m
- $n_r$ complex refractive index of smoke particles
- $K$ extinction coefficient, m$^{-1}$
- $D$ optical density per meter, m$^{-1}$
- $K_m$ specific extinction coefficient, m$^2$/g
- $D_p$ specific optical density (–)
- $D_m$ mass optical density, m$^2$/g
- $I_\lambda$ intensity of light at wavelength $\lambda$
- $B$ luminance
- $C$ contrast
- $s$ visibility range, m
- $L$ pathlength
- $\Gamma$ coagulation coefficient, cm$^3$/s
- $t$ time
- $\Delta M$ mass loss of sample, g
- $P$ detector output, volts
- $R(d)$ detector size response function, $\mu$V cm$^{-1}$
- $V_c$ volume of chamber, m$^3$
- $A$ area of sample, m$^2$
- $M_s$ mass of smoke, g

REFERENCES CITED